ABSRACT OF LECTURE IN HONOR OF RAMIREZ ARELLANO
title: **SOLUTIONS OF PDE'S AND RESOLUTIONS OF SIGULARITIES**

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This lecture is concerned with the relationship between the operator $\bar{\partial}$ and the study of the singularities of complex analytic varieties. Let $V \subset \mathbb{C}^n$ be the variety defined by $h_1 = \cdots = h_n = 0$ wit the h_i holomorphic functions in a neighborhood of the origin in $\mathbb{C}^n$. Let r be the function in a neighborhood of the origin in $\mathbb{C}^{n+1}$ defined by:

$$r(z_1, \ldots, z_{n+1}) = Re(z_{n+1}) + \sum_1^n |h_\ell(z_1, \ldots, z_n)|^2.$$

Let $M' \subset \mathbb{C}^{n+1}$ be a comlex analytic manifold with a smooth boundary M so that, in a neighborhood of $P \in M$ P, the origin of $\mathbb{C}^{n+1}$, M is given by $r = 0$ and M' by $r \geq 0$. Let $u \in L_2(M)$ and le α be $(0,1)$-form with square integrable coefficients. Suppose that u is orthogonal to the space of square integrable functions and that $\bar{\partial}_b u = \alpha$. Then, near the origin, u is always "smoother" then α if and only if $V = 0$. The talk is concerned with ways to study this behavior by analyzing the singularity of V at 0. D'Angelo developed a theory of type of holomorphic curves through the origin with $r = 0$ in which the type is ∞ if and only if $dimV = 0$ and Catlin proved that the above u is always "smoother" then α if and only if the D'Angelo order is finite. More precisely smoothness at 0 is measured by Sobolev norms of germs at 0, so that it means that there exists $\varepsilon > 0$ so that

$$\|\alpha\|_s < \infty \Rightarrow \|u\|_{s+\varepsilon} < \infty.$$

Another approach to this question is via the ideal type. That is if $z : \mathbb{C} \to \mathbb{C}^{n+1}$, $z(t) = (z_1(t), \ldots, z_{n+1}(t))$, is a germ off a holomorphic curve at 0 in V we have: $h_\ell(z(t)) = 0$ for small t. Hence $z_{n+1}(t) = 0$ and

$$\frac{\partial h_\ell}{\partial t} = \sum_i^n h_{\ell z_i} z_i'(t) = 0$$

for t near 0. Hence $det\big(h_{\ell z_i}\big)(t) = 0$. Terefore $1 \neq \big(det(h_{\ell z_i})\big)$ and $1 \neq \sqrt[k]{\big(det(h_{\ell z_i})\big)}$ for $k \in \mathbb{Z}^+$. So that starting with the ideal $I^1 = \big(det(h_{\ell z_i})\big)$ we construct a set of ideals: $\{I^{a_1,b_1,\ldots,a_N,b_N}\}$ by successively taking a_i Jacobians and b_i roots. Then $dimV > 0$ if and only if one of these ideals contains 1. If $f \in I^{a_1,b_1,\ldots,a_N,b_N}$ then f is a subelliptic multiplier, in the sense that $\|fu\|_{s+\varepsilon} \leq C_s\|\alpha\|_s$ where $\varepsilon > 0$ depends on $a_1, \ldots .b_N$. Siu and Kim- Zaitsev proved that $dimV = 0$ there exists an effective algorithm (that is, a choice of the a's and b's with $1 \in I^{a_1,b_1,\ldots,a_N,b_N}$ such that N depends only on the D'Angelo type. Here we will go beyond subelliptic estimates to discuss Hölder and L_p estimates for this we will need optimal microlocal estimates. We will discuss these and analyze the special case $h_\ell(z_1, \ldots, z_n) = g_{\ell-1}(z_1, \ldots, z_{\ell-1}) + f_\ell(z_\ell)$ for $\ell = 1, \ldots, n$ and $g_0 = 0$.